

## ANALYSIS OF FINLINE DISCONTINUITIES

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## ABSTRACT

Finline discontinuities are analyzed by means of mode matching technique and spectral domain technique. Numerical results are given.

## INTRODUCTION

With increasing activities in millimeter wave field, more attention was paid to the finline technique for development of millimeter-wave integrated circuits. During the last decade a great many of work have been done to analyze the dispersion characteristics and impedance of finlines [1][2][3]. On the contrary, there are relatively few papers concerning the discontinuities of finlines. It is necessary to know the equivalent parameters of finline discontinuities in computer-aided-design of active and passive microwave circuits. Hennawy and Schünemann [4] analyzed finline discontinuities such as steps, notches and patches by using mode analysis. Sorrentino and Itoh [5] evaluated the discontinuity by carrying out transverse resonance analysis. Helard, Citerne et al [6] have treated finline single and multiple steps by using mode matching technique and spectral domain method.

In present paper, the finline notch and patch are analyzed by a method similar to [6]. But symmetry of the configuration is taken advantage of so that the evaluation is labor-saving.

## EQUIVALENT CIRCUITS OF NOTCH AND PATCH

The structures to be investigated are shown

in Fig.1 in which (b) is a finline notch and (c) is a finline patch. It is noticed that they are both symmetrical structure. However, there is no limitation on the locations of finline slots. Usually, a notch is replaced by a  $\pi$  network and a patch by a T network.

For a symmetrical two port network the symmetrical plane becomes a magnetic wall in the case of even mode excitation and an electric wall in case of odd mode excitation. Hence only one half section of the network is needed to be considered as shown in Fig. 3. The scattering parameters of the original two port network can then be expressed as:

$$S_{11} = S_{22} = \frac{1}{2}(S_{11}^e + S_{11}^o) \quad (1)$$

$$S_{12} = S_{21} = \frac{1}{2}(S_{11}^e - S_{11}^o) \quad (2)$$

where  $S_{11}^e$  (or  $S_{11}^o$ ) is the reflection coefficient of even-mode (or odd-mode) half-section network.

## FORMULATION OF PROBLEM

Model analysis is carried out at first. The transverse fields at the left of junction plane which is the interface between waveguide I and II are expressed in terms of mode functions.

$$\vec{E}_T^I = \sum_{i=1}^M (V_i^{I+} + V_i^{I-}) \vec{e}_{iT}^I \quad (3)$$

$$\vec{H}_T^I = \sum_{i=1}^M (V_i^{I+} - V_i^{I-}) \vec{h}_{iT}^I \quad (4)$$

where  $V_i^{I+}$  is the  $i$ th mode voltage of forward wave,

$V_i^{I-}$  is the  $i$ th mode voltage of backward wave,

$\vec{e}_{iT}^I$  is the  $i$ th mode normalized transverse electric field mode function,

$\vec{h}_{iT}^I$  is the  $i$ th mode normalized transverse magnetic field mode function.

For even-mode excitation, the fields in waveguide II can be expressed as:

$$\vec{E}_T^{II} = \sum_{j=1}^N V_j \cos \beta_j^{II} l \vec{e}_{jT}^{II} \quad (5)$$

$$\vec{H}_T^{II} = \sum_{j=1}^N V_j \sin \beta_j^{II} l \vec{h}_{jT}^{II} \quad (6)$$

where  $\beta_j^{II}$  is the propagating constant of the  $j$ th mode in waveguide II.  $l$  is the half width of the patch or notch.

Employing the fields continuity condition at junction plane, we have

$$\sum_{i=1}^M (V_i^{I+} + V_i^{I-}) \vec{e}_{iT}^I = \sum_{j=1}^N V_j \cos \beta_j^{II} l \vec{e}_{jT}^{II} \quad (7)$$

$$\sum_{i=1}^M (V_i^{I+} - V_i^{I-}) \vec{h}_{iT}^I = \sum_{j=1}^N V_j \sin \beta_j^{II} l \vec{h}_{jT}^{II} \quad (8)$$

Taking cross product of (7) with  $\vec{h}_{jT}^{II*}$  ( $j=1,2, \dots, N$ ) and integrating over waveguide II yield the relationship between  $V^{I+}$ ,  $V^{I-}$  and  $V^{II}$ .  $\vec{h}_{jT}^{II*}$  is the conjugate function of  $\vec{h}_{jT}^{II}$ . Similarly inner product should be carried out for (8) with  $\vec{e}_{iT}^{I*}$ . In the above processes the orthogonal condition of mode functions have been used. Consequently, we have

$$[Q]([\vec{V}^{I+}] + [\vec{V}^{I-}]) = [R^e][\vec{V}^{II}] \quad (9)$$

$$[H]([\vec{V}^{I+}] - [\vec{V}^{I-}]) = [T^e][\vec{V}^{II}] \quad (10)$$

where  $[Q]$  is  $N \times M$  matrix and its element is  $Q_{uv}$

$$Q_{uv} = \int_{s_{II}} \vec{e}_{vT}^I \times \vec{h}_{uT}^{II*} \cdot \vec{z} \, ds$$

$[R]$  is  $N \times N$  diagonal matrix and its element is

$$R_{uu}^e = s_u \cos \beta_u^{II} l \beta_u^{II} / |\beta_u^{II}|$$

$$s_u = \begin{cases} 1, & \text{if } (\beta_u)^2 > 0; \\ -1, & \text{if } (\beta_u)^2 \leq 0. \end{cases}$$

$[H]$  is  $M \times M$  diagonal matrix and its element is  $H_{uu}$ ,

$$H_{uu} = s_u \beta_u^{I*} / |\beta_u^I|$$

$\beta_u^I$  is the propagating constant of the  $u$ th mode in waveguide I.

$[T^e]$  is  $M \times N$  matrix and its element is  $T_{uv}^e$ ,

$$T_{uv}^e = j \sin \beta_v^{II} l \int_{s_I} \vec{e}_{uT}^{I*} \times \vec{h}_{vT}^{II} \cdot \vec{z} \, ds$$

$$[\vec{V}^{I+}] = [V_1^{I+}, \dots, V_M^{I+}]^T$$

$$[\vec{V}^{I-}] = [V_1^{I-}, \dots, V_M^{I-}]^T$$

$$[\vec{V}^{II}] = [V_1^{II}, \dots, V_N^{II}]^T$$

After some algebraic manipulation  $[\vec{V}^{II}]$  is eliminated and the relationship between forward waves and backward waves is obtained.

$$[\vec{V}^{I-}] = ([H] + [T^e][R^e]^{-1}[Q])^{-1}([H] - [T^e][R^e]^{-1}[Q])[\vec{V}^{I+}] = [S^e][\vec{V}^{I+}] \quad (11)$$

$S_{11}^e$  can then be obtained from  $[S^e]$ .

For odd mode excitation, the process is similar to even mode case and the final result is

$$[\vec{V}^{I-}] = ([H] + [T^o][R^o]^{-1}[Q])^{-1}([H] - [T^o][R^o]^{-1}[Q])[\vec{V}^{I+}] = [S^o][\vec{V}^{I+}] \quad (12)$$

where  $[T^o]$  is  $M \times N$  matrix and its element  $T_{uv}^o$  is

$$T_{uv}^o = \cos \beta_v^{II} l \int_{s_I} \vec{e}_{uT}^{I*} \times \vec{h}_{vT}^{II} \cdot \vec{z} \, ds$$

$[R^o]$  is  $N \times N$  diagonal matrix and its element is  $R_{uu}^o$ .

$$R_{uu}^o = j s_u \sin \beta_u^{II} l \beta_u^{II} / |\beta_u^{II}|$$

From equation (12)  $S_{11}^o$  can be deduced as before.

## SPECTRAL DOMAIN TECHNIQUE

The propagating constants of dominant and high-order modes are usually needed. The characteristic equation is obtained by employing spectral domain immittance approach as usual.

$$\begin{bmatrix} \tilde{Y}_{xx} & \tilde{Y}_{xz} \\ \tilde{Y}_{zx} & \tilde{Y}_{zz} \end{bmatrix} \begin{bmatrix} \tilde{E}_x \\ \tilde{E}_z \end{bmatrix} = \begin{bmatrix} \tilde{J}_x \\ \tilde{J}_z \end{bmatrix}$$

where  $\tilde{E}_x$ ,  $\tilde{E}_z$  are Fourier transformations of slot fields.  $\tilde{J}_x$ ,  $\tilde{J}_z$  are Fourier transformation of surface currents.  $\tilde{Y}_{xx}$ ,  $\tilde{Y}_{xz}$  ... etc. are dyadic Green's functions in spectral domain. The  $E_x$  and  $E_z$  can be expressed as follows:

$$E_x = \sum_{i=1}^M c_i \xi_{xi}, \quad E_z = \sum_{j=1}^N d_j \eta_{zj}$$

where  $\xi_{xi}$  and  $\eta_{zj}$  are the basis functions of slot electric fields in the direction of x- and z-axis respectively.

By Galerkin's method, we obtain M+N linear simultaneous equations of  $c_i$  and  $d_j$ . The propagation constants are calculated one by one from the equation which is derived by putting the matrix determinant equal to zero.

The integrations encountered previously may be carried out by Parseval's theorem. For example, the element of the matrix  $[Q]$  is  $Q_{uv}$  as follows.

$$\begin{aligned} Q_{uv} &= \int_{s_{II}} \tilde{e}_{vT}^I \times \tilde{h}_{uT}^{*} \cdot \vec{z} \, ds \\ &= \frac{1}{2b} \sum_{n=-\infty}^{\infty} \int_0^{h_1+D+h_2} F(y) \, dy \end{aligned}$$

where

$$F(y) = \tilde{E}_{xv}(\alpha_n, y) \tilde{H}_{yu}^{*}(\alpha_n, y) - \tilde{E}_{yv}(\alpha_n, y) \tilde{H}_{xu}^{*}(\alpha_n, y)$$

Before integration the finline fields must be known. The solution of finline fields in spectral domain are as follows:

$$\begin{aligned} \tilde{E}_{1z}(\alpha_n, y) &= A^e(\alpha_n) \sinh \gamma_1 (h_1 + h_2 + D - y) \\ \tilde{E}_{2z}(\alpha_n, y) &= B^e(\alpha_n) \sinh \gamma_2 (h_1 + D - y) \\ &\quad + C^e(\alpha_n) \cosh \gamma_2 (h_1 + D - y) \end{aligned}$$

$$\tilde{E}_{3z}(\alpha_n, y) = D^e(\alpha_n) \sinh \gamma_1 y$$

$$\tilde{H}_{1z}(\alpha_n, y) = A^h(\alpha_n) \cosh \gamma_1 (h_1 + h_2 + D - y)$$

$$\tilde{H}_{2z}(\alpha_n, y) = B^h(\alpha_n) \sinh \gamma_2 (h_1 + D - y)$$

$$+ C^h(\alpha_n) \cosh \gamma_2 (h_1 + D - y)$$

$$\tilde{H}_{3z}(\alpha_n, y) = D^h(\alpha_n) \cosh \gamma_1 y$$

The other field components can be derived from  $\tilde{E}_z$  and  $\tilde{H}_z$ . Instead of extensive algebraic manipulation a convenient matrix technique [2] is used to solve the coefficients  $A^e, B^e, \dots, C^h, D^h$ . The following equation satisfying the boundary conditions gives out the values of coefficients.

$$[M_E][U] = [\tilde{E}]$$

where

$$[U] = [A^e, B^e, \dots, C^h, D^h]^T$$

$$[\tilde{E}] = [0, 0, \dots, \tilde{E}_x, \tilde{E}_z]^T$$

The above equation can be solved once  $\beta$ 's and slot field of interested mode are found.

## NUMERICAL RESULTS

The notch and patch inside WR-28 waveguide are analyzed and the calculated equivalent parameters are given in Fig.4.

## CONCLUSION

The combination of mode-matching technique and spectral domain technique serve as a powerful tool to treat finline discontinuities problems such as finline notch and patch. Numerical results are given. The authors believe that the results obtained by present method will find wide application to millimeter wave band filter designs.

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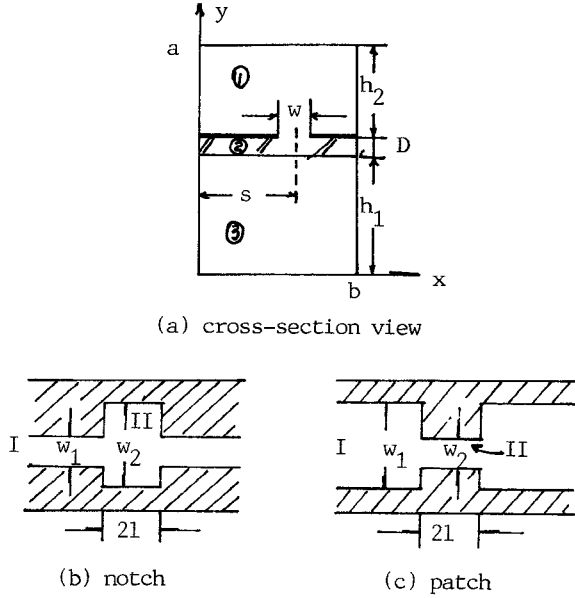


Fig.1 Discontinuities in finline

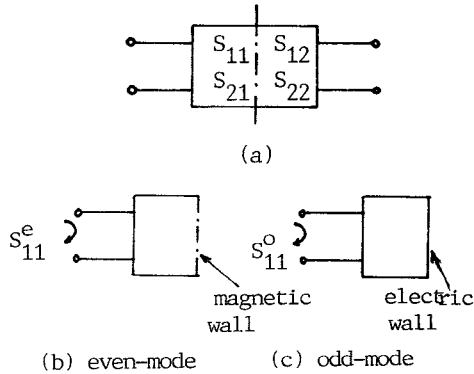


Fig. 3

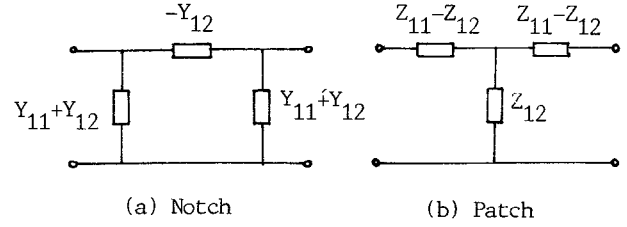


Fig.2 Equivalent circuits

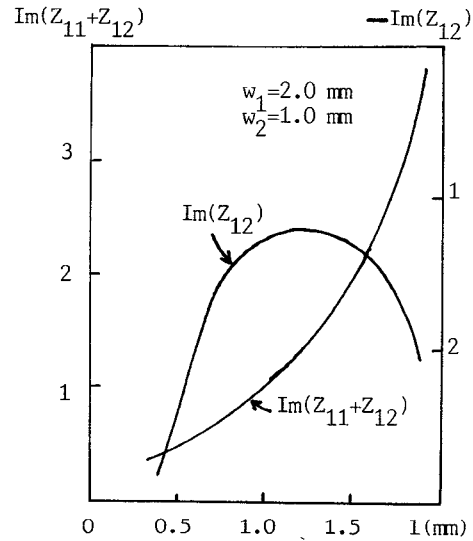
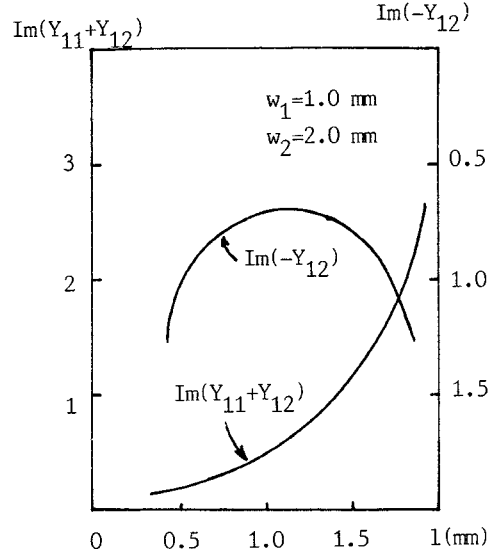


Fig.4 Calculated results of a notch and a patch.  
 $f=35$  GHz,  $D=.254$  mm,  $\epsilon_r=2.22$ ,  
 $s=1.778$  mm,  $h_1=h_2=3.429$  mm.